ELSEVIER

Journal of Geometry and Physics 40 (2002) 370–378

JOURNAL OF GEOMETRY AND PHYSICS

www.elsevier.com/locate/jgp

Space-like submanifolds in the de Sitter spaces

Liu Ximin

Department of Applied Mathematics, Dalian University of Technology, Dalian 116024, China

Received 15 January 2001; received in revised form 5 April 2001

Abstract

In this paper, by using Cheng–Yau's self-adjoint operator $\Box$, we study the space-like submanifolds in the de Sitter spaces and obtain some general rigidity results.

MSC: 53C40; 53C42; 53C50

Subj. Class.: Differential geometry; General relativity

Keywords: Space-like submanifold; de Sitter space; Totally umbilical submanifold

1. Introduction

Let $M_p^{n+p}(c)$ be an $(n+p)$-dimensional connected semi-Riemannian manifold of constant curvature c whose index is p. It is called an indefinite space form of index p and simply a space form when $p = 0$. If $c > 0$, we call it as a de Sitter space of index p, denote it by $S_p^{n+p}(c)$. The study of space-like hypersurfaces in de Sitter space has been recently of substantial interest from both physics and mathematical point of view. Akutagawa [2] and Ramanathan [15] investigated space-like hypersurfaces in a de Sitter space and proved independently that a complete space-like hypersurface in a de Sitter space with constant mean curvature is totally umbilical if the mean curvature H satisfies $H^2 \leq c$ when $n = 2$ and $n^2H^2 < 4(n-1)c$ when $n \geq 3$. Later, Cheng [5] generalized this result to general submanifolds in a de Sitter space.

On the other hand, Cheng and Ishikawa [6] have recently shown that the totally umbilical round spheres are the only compact space-like hypersurfaces in $S_1^{n+1}(1)$ with constant scalar curvature $S < n(n-1)$. Some other authors, such as Ximin [11] and Zheng [16,17], have also obtained interesting results related to the characterization of the totally umbilical round

E-mail address: xmliu@dlut.edu.cn (L. Ximin).

PII: S0393-0440(01)00042-0

spheres as the only compact space-like hypersurfaces in the de Sitter space with constant scalar curvature.

In [10], Li studied the Cheng–Yau's self-adjoint operator $\Box$ for a given codazzi tensor field $\phi = \sum_{i,j}\phi_{ij}\omega_i\omega_j$ on an n-dimensional compact Riemannian manifold and obtain a general rigidity theorem which generalized Cheng–Yau's work [8]. By using this result he studied the space-like hypersurfaces in Lorentzian space form and obtained some rigidity theorems which naturally generalize the existing results of Akutagawa [2], Cheng and Yau [7], Ramanathan [15] and Montiel [12] about Goddard's conjecture [9].

In the present paper, we would like to use Cheng–Yau's self-adjoint operator $\Box$ to study the space-like submanifolds in de Sitter space and obtain some general rigidity results.

2. Cheng–Yau's self-adjoint operator $\Box$

In this section, we review the fundamental results about the Cheng–Yau's self-adjoint operator $\Box$, for details see [10].

Let M^n be an n-dimensional Riemannian manifold, $e_1, \dots, e_n$ a local orthonormal frame field on M^n, and let $\omega_1, \dots, \omega_n$ be its dual coframe field. Then, the structure equations of M^n are given by

$$d\omega_i = \sum_j \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0, \tag{1}$$

$$d\omega_{ij} = \sum_k \omega_{ik} \wedge \omega_{kj} - \frac{1}{2}\sum_{k,l} R_{ijkl}\omega_k \wedge \omega_l, \tag{2}$$

where ω_{ij} is the Levi-Civita connection form and R_{ijkl} are the components of the curvature tensor of M^n.

For any C^2-function f defined on M^n, we defined its gradient and Hessian by the following formulas:

$$d f = \sum_i f_i\omega_i, \tag{3}$$

$$\sum_j f_{ij}\omega_j = d f_i + \sum_j f_j\omega_{ji}. \tag{4}$$

Let $\phi = \sum_{ij}\phi_{ij}\omega_i \otimes \omega_j$ be a symmetric tensor defined on M^n. The covariant derivative of ϕ_{ij} is defined by

$$\sum_k \phi_{ijk}\omega_k = d\phi_{ij} + \sum_k \phi_{kj}\omega_{ki} + \sum_k \phi_{ik}\omega_{kj}. \tag{5}$$

We call the symmetric tensor $\phi = \sum_{i,j}\phi_{ij}\omega_i \otimes \omega_j$ a Codazzi tensor if $\phi_{ijk} = \phi_{ikj}$.

The second covariant derivative of ϕ_{ij} is defined by

$$\sum_l \phi_{ijkl}\omega_l = d\phi_{ijk} + \sum_m \phi_{mjk}\omega_{mi} + \sum_m \phi_{imk}\omega_{mj} + \sum_m \phi_{ijm}\omega_{mk}. \tag{6}$$

Then, we have the following Ricci identities:

$$\phi_{ijkl} - \phi_{ijlk} = \sum_m \phi_{mj} R_{mikl} + \sum_m \phi_{im} R_{mjkl}. \tag{7}$$

The definition of the following self-adjoint operator $\square$ was first introduced by Cheng–Yau in [8].

Definition 2.1. Let $\phi = \sum_{i,j} \omega_i \otimes \omega_j$ be a Codazzi tensor field on a Riemanian manifold M^n. We define the operator $\square$ associated to ϕ by

$$\square f = \sum_{i,j} \left(\left(\sum_k \phi_{kk} \right) \delta_{ij} - \phi_{ij} \right) f_{ij} \tag{8}$$

for any C^2-function f defined on M^n.

The Laplacian $\Delta \phi_{ij}$ of the tensor ϕ_{ij} is defined to be $\sum_k \phi_{ijkk}$, and we have [10]:

$$\Delta \phi_{ij} = \left(\sum_k \phi_{kk} \right)_{ij} + \sum_{m,k} \phi_{mk} R_{mijk} + \sum_{m,k} \phi_{im} R_{mkjk} \tag{9}$$

Let $|\phi|^2 = \sum_{i,j} \phi_{ij}^2$, $|\nabla \phi|^2 = \sum_{i,j,k} \phi_{ijk}^2$ and $\mathrm{tr}\, \phi = \sum_k \phi_{kk}$. Then, from (9) we have

$$\frac{1}{2} \Delta |\phi|^2 = |\nabla \phi|^2 + \sum_{i,j} \phi_{ij} (\mathrm{tr}\, \phi)_{ij} + \sum_{i,j,m,k} \phi_{ij} \phi_{mk} R_{mijk} + \sum_{i,j,m,k} \phi_{ij} \phi_{im} R_{mkjk}. \tag{10}$$

Near a given point $p \in M^n$, we choose a local orthonormal frame field $\{e_1, \ldots, e_n\}$ and its dual frame field $\{\omega_1, \ldots, \omega_n\}$ such that $\phi = \sum_{i,j} \phi_{ij} \omega_i \otimes \omega_j$, $\phi_{ij} = \lambda_i \delta_{ij}$ at p. Then (10) is simplified to

$$\frac{1}{2} \Delta |\phi|^2 = |\nabla \phi|^2 + \sum_i \lambda_i (\mathrm{tr}\, \phi)_{ii} + \frac{1}{2} \sum_{i,j} R_{ijij} (\lambda_i - \lambda_j)^2. \tag{11}$$

Denoting the second symmetric function of ϕ_{ij} by m, we have

$$m = \sum_{i \neq j} \lambda_i \lambda_j = (\mathrm{tr}\, \phi)^2 - |\phi|^2. \tag{12}$$

From (11) and (12), we have

$$\frac{1}{2} \Delta (\mathrm{tr}\, \phi)^2 = \frac{1}{2} \Delta m + |\nabla \phi|^2 + \sum_i \lambda_i (\mathrm{tr}\, \phi)_{ii} + \frac{1}{2} \sum_{i,j} R_{ijij} (\lambda_i - \lambda_j)^2. \tag{13}$$

From (13) and (8), we have

$$\square (\mathrm{tr}\, \phi) = \frac{1}{2} \Delta m + |\nabla \phi|^2 - |\nabla (\mathrm{tr}\, \phi)|^2 + \frac{1}{2} \sum_{i,j} R_{ijij} (\lambda_i - \lambda_j)^2. \tag{14}$$

Since $\Box$ is self-adjoint and M^n is compact, we obtain by integration of (14)

$$\int_{M^n} [|\nabla\phi|^2 - |\nabla(\operatorname{tr}\phi)|^2] + \int_{M^n} \frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2 = 0. \tag{15}$$

So we have the following theorem.

Theorem 2.1 (Li [10]). *Let $\phi = \sum_{i,j}\omega_i \otimes \omega_j$ be a Codazzi tensor field on a Riemanian manifold M^n. We assume the following condition:*

$$|\nabla\phi|^2 \geq |\nabla(\operatorname{tr}\phi)|^2. \tag{16}$$

1. *If M^n has positive sectional curvature, then all the eigenvalues of ϕ_{ij} are the same on M^n.*
2. *If M^n has non-negative sectional curvature, then we have $|\nabla\phi|^2 = |\nabla(\operatorname{tr}\phi)|^2$ and $R_{ijij} = 0$, when $\lambda_i \neq \lambda_j$ on M^n.*

Okumuru [14] established the following lemma (see also [3]).

Lemma 2.1. *Let μ_i, $i = 1, \dots, n$, be real numbers such that $\sum_i \mu_i = 0$ and $\sum_i \mu_i^2 = \beta^2$, where $\beta = \text{constant} \geq 0$. Then*

$$-\frac{n-2}{\sqrt{n(n-1)}}\beta^3 \leq \sum_i \mu_i^3 \leq \frac{n-2}{\sqrt{n(n-1)}}\beta^3, \tag{17}$$

and the equality holds in (17) if and only if at least $(n-1)$ of the μ_i are equal.

3. Space-like submanifolds in de Sitter space

Let M^n be an n-dimensional space-like submanifold in $S_p^{n+p}(c)$. We choose a local field of semi-Riemannian orthonormal frames $e_1, \dots, e_{n+p}$ in $S_p^{n+p}(c)$ such that at each point of M^n, $e_1, \dots, e_n$ span the tangent space of M^n and form an orthonormal frame there. We use the following convention on the range of indices:

$$1 \leq A, B, C, \dots \leq n+p, \qquad 1 \leq i, j, k, \dots \leq n, \qquad n+1 \leq \alpha, \beta, \gamma \leq n+p.$$

Let $\omega_1, \dots, \omega_{n+p}$ be its dual frame field so that the semi-Riemannian metric of $S_p^{n+p}(c)$ is given by $d\bar{s}^2 = \sum_i \omega_i^2 - \sum_\alpha \omega_\alpha^2 = \sum_A \epsilon_A \omega_A^2$, where $\epsilon_i = 1$ and $\epsilon_\alpha = -1$. Then, the structure equations of $S_p^{n+p}(c)$ are given by

$$d\omega_A = \sum_B \epsilon_B \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0, \tag{18}$$

$$d\omega_{AB} = \sum_C \epsilon_C \omega_{AC} \wedge \omega_{CB} - \frac{1}{2}\sum_{C,D} K_{ABCD}\omega_C \wedge \omega_D, \tag{19}$$

$$K_{ABCD} = c\epsilon_A\epsilon_B(\delta_{AC}\delta_{BD} - \delta_{AD}\delta_{BC}). \tag{20}$$

Restricting these forms to M^n, we have

$$\omega_\alpha = 0, \quad n+1 \le \alpha \le n+p. \tag{21}$$

From Cartan's lemma, we can write

$$\omega_{\alpha i} = \sum_j h_{ij}^\alpha \omega_j, \quad h_{ij}^\alpha = h_{ji}^\alpha. \tag{22}$$

From these formulas, we obtain the structure equations of M^n:

$$\mathrm{d}\omega_i = \sum_j \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0, \tag{23}$$

$$\mathrm{d}\omega_{ij} = \sum_k \omega_{ik} \wedge \omega_{kj} - \frac{1}{2}\sum_{k,l} R_{ijkl}\omega_k \wedge \omega_l, \tag{24}$$

$$R_{ijkl} = c(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) - \sum_\alpha (h_{ik}^\alpha h_{jl}^\alpha - h_{il}^\alpha h_{jk}^\alpha), \tag{25}$$

where R_{ijkl} are the components of the curvature tensor of M^n and

$$h = \sum_\alpha h_\alpha e_\alpha = \sum_{i,j,\alpha} h_{ij}^\alpha \omega_i \otimes \omega_j \otimes e_\alpha \tag{26}$$

is the second fundamental form of M^n.

For indefinite Riemannian manifolds in detail, refer to O'Neill [13].

Denote $L_\alpha = (h_{ij}^\alpha)_{n\times n}$ and $H_\alpha = (1/n)\sum_i h_{ii}^\alpha$ for $\alpha = n+1, \ldots, n+p$. Then, the mean curvature vector field ξ, the mean curvature H and the square of the length of the second fundamental form S are expressed as

$$\xi = \sum_\alpha H_\alpha e_\alpha, \qquad H = |\xi|, \qquad S = \sum_{\alpha,i,j} (h_{ij}^\alpha)^2, \tag{27}$$

respectively. Moreover, the normal curvature tensor $\{R_{\alpha\beta kl}\}$ and the normalized scalar curvature R are expressed as

$$R_{\alpha\beta kl} = \sum_m (h_{km}^\alpha h_{ml}^\beta - h_{lm}^\alpha h_{mk}^\beta), \qquad R = c + \frac{1}{n(n-1)}(S - n^2H^2). \tag{28}$$

If $R_{\alpha\beta kl} = 0$ at point x of M^n, we say that the normal connection of M^n is flat at x and it is well known [4] that $R_{\alpha\beta kl} = 0$ at x if and only if h_α are simultaneously diagonalizable at x.

Suppose $H > 0$ on M^n and choose $e_{n+1} = \xi/H$. Then, it follows that

$$H_{n+1} = H, \qquad H_\alpha = 0, \quad \alpha > n+1. \tag{29}$$

Suppose that the normal bundle of M^n is flat, then we can choose $e_1, \ldots, e_n$ such that

$$h_{ij}^\alpha = \lambda_i^\alpha \delta_{ij}, \quad \alpha = n+1, \ldots, n+p. \tag{30}$$

Let $\phi_{ij} = h^{\alpha}_{ij}$. From (15) we have

$$\int_{M^n} [|\nabla h|^2 - n^2|\nabla H|^2] + \int_{M^n} \frac{1}{2}\sum_{i,j,\alpha} R_{ijij}(\lambda_i^{\alpha} - \lambda_j^{\alpha})^2 = 0. \tag{31}$$

By using the Gauss equation, we have

$$\frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i^{n+1} - \lambda_j^{n+1})^2 = ncS_{n+1} - n^2H^2c + S_{n+1}^2 - nH\sum_i (\lambda_i^{n+1})^3, \tag{32}$$

where $S_{n+1} = \sum_{i,j}(h_{ij}^{n+1})^2$.

Let $\mu_i^{n+1} = \lambda_i^{n+1} - H$ and $|Z|^2 = \sum_i (\mu_i^{n+1})^2$. We have

$$\sum_i \mu_i^{n+1} = 0, \qquad |Z|^2 = S_{n+1} = nH^2, \tag{33}$$

$$\sum_i (\lambda_i^{n+1})^3 = \sum_i (\mu_i^{n+1})^3 + 3H|Z|^2 - nH^3, \tag{34}$$

Putting (33) and (34) into (32), we get

$$\frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i^{n+1} - \lambda_j^{n+1})^2 = |Z|^2(nc - nH^2 + |Z|^2) - nH\sum_i (\mu_i^{n+1})^3. \tag{35}$$

By using Lemma 2.1, we have

$$\frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i^{\alpha} - \lambda_j^{\alpha})^2 \geq (S_{n+1} - nH^2)\left(nc - 2nH^2 + S_{n+1} - \frac{n(n-2)}{\sqrt{n(n-1)}} H\sqrt{S_{n+1} - nH^2}\right). \tag{36}$$

Putting (36) into (31), we have

$$\int_{M^n}\left[(|\nabla h|^2 - n^2|\nabla H|^2) + (S_{n+1} - nH^2) \times \left(nc - 2nH^2 + S_{n+1} - \frac{n(n-2)}{\sqrt{n(n-1)}} H\sqrt{S - nH^2}\right)\right] \leq 0. \tag{37}$$

Note that

$$nc - 2nH^2 + S_{n+1} - \frac{n(n-2)}{\sqrt{n(n-1)}} H\sqrt{S_{n+1} - nH^2} = \left(\sqrt{S_{n+1} - nH^2} + \frac{1}{2}(n-2)H\sqrt{\frac{n}{n-1}}\right)^2 + n\left(c - \frac{n^2}{4(n-1)}H^2\right). \tag{38}$$

From (37) and (38), we have the following theorem.

Theorem 3.1. *Let M^n be an n-dimensional compact space-like submanifold in an $(n+p)$-dimensional de Sitter space $S_p^{n+p}(c)$. Suppose that M^n has flat normal bundle, if*

$$|\nabla h|^2 \geq n^2|\nabla H|^2 \tag{39}$$

and

$$H^2 < 4\frac{(n-1)c}{n^2}, \tag{40}$$

then $S_{n+1} \equiv nH^2$ and M^n is a pseudo-umbilical submanifold.

Corollary 3.1. *Let M^n be an n-dimensional compact space-like submanifold with constant scalar curvature R in an $(n+p)$-dimensional de Sitter space $S_p^{n+p}(c)$. Suppose that M^n has flat normal bundle, if $R-c \leq 0$ and*

$$H^2 < 4\frac{(n-1)c}{n^2}, \tag{41}$$

then $S_{n+1} \equiv nH^2$ and M^n is a pseudo-umbilical submanifold.

Proof. From (28), we have $n^2H^2 - S = n(n-1)(c-R) \geq 0$. Taking the covariant derivative on both sides of this equality, we get

$$n^2HH_k = \sum_{i,j,\alpha} h_{ij}^{\alpha} h_{ijk}^{\alpha}, \quad k = 1, \ldots, n.$$

For every k, it follows from Cauchy–Schwarz's inequality that

$$n^4H^2H_k^2 = \left(\sum_{i,j,\alpha} h_{ij}^{\alpha} h_{ijk}^{\alpha}\right)^2 \leq S\sum_{i,j,\alpha}(h_{ijk}^{\alpha})^2. \tag{42}$$

Taking sum on both sides of (42) with respect to k, we have

$$n^4H^2|\nabla H|^2 = n^4H^2\sum_k H_k^2 \leq S\sum_{(i,j,k,\alpha)}(h_{ijk}^{\alpha})^2 \leq n^2H^2\sum_{(i,j,k,\alpha)}(h_{ijk}^{\alpha})^2, \tag{43}$$

i.e. $|\nabla h|^2 \geq n^2|\nabla H|^2$, then Corollary 3.1 follows from Theorem 3.1. □

Now consider the quadratic form $Q(u,t) = u^2 - ((n-2)/\sqrt{n-1})ut - t^2$. By the orthogonal transformation

$$\bar{u} = \frac{1}{\sqrt{2n}}\{(1+\sqrt{n-1})u + (1-\sqrt{n-1})t\},$$
$$\bar{t} = \frac{1}{\sqrt{2n}}\{(\sqrt{n-1}-1)u + (\sqrt{n-1}+1)t\}.$$

$Q(u,t)$ turns into $Q(u,t) = (n/2\sqrt{n-1})(\bar{u}^2 - \bar{t}^2)$, where $\bar{u}^2 + \bar{t}^2 = u^2 + t^2$.

Take $u=\sqrt{\bar S_{n+1}}$, $t=\sqrt{n}H$, then

$$nc-nH^2-n(n-2)H\sqrt{\frac{\bar S_{n+1}}{n(n-1)}}+\bar S_{n+1}=nc+Q(u,t)$$
$$=nc+\frac{n(\bar u^2-\bar t^2)}{2\sqrt{n-1}}=nc+\frac{n(-\bar u^2-\bar t^2)}{2\sqrt{n-1}}+\frac{n\bar u^2}{\sqrt{n-1}}\geq nc-\frac{n\bar S_{n+1}}{2\sqrt{n-1}}. \tag{44}$$

From (44) and (37) we have

$$0\geq\int_{M^n}(|\nabla h|^2-n^2|\nabla H|^2)+(S_{n+1}-nH^2)\left[nc-\frac{n(S_{n+1}-nH^2)}{2\sqrt{n-1}}\right]. \tag{45}$$

Therefore, we have the following theorem.

Theorem 3.2. *Let $M^n (n\geq 3)$ be a compact space-like submanifold in the de Sitter space $S_p^{n+p}(c)$. Suppose that $|\nabla h|^2\geq n^2|\nabla H|^2$. If M^n has flat normal bundle and*

$$S_{n+1}<nH^2+2\sqrt{n-1}c, \tag{46}$$

then $S_{n+1}=nH^2$ and M^n is a pseudo-umbilical submanifold.

Theorem 3.3. *Let M^n be an n-dimensional compact space-like submanifold with constant scalar curvature and with non-negative sectional curvature in $S_n^{n+p}(c)$. Suppose that M^n has flat normal bundle, if the normalized mean curvature vector is parallel and R satisfies $R<c$, then M^n is totally umbilical.*

Proof. We have from (28),

$$n^2H^2-\|h\|^2=n(n-1)(c-R), \tag{47}$$

where R is the normalized scalar curvature of M^n. Taking the covariant derivative of (47) and using the fact that $R=$ constant, we obtain

$$n^2HH_k=\sum_{i,j,\alpha}h_{ij}^\alpha\cdot h_{ijk}^\alpha,$$

and hence by Cauchy–Schwarz inequality, we have

$$\sum_k n^4H^2(H_k)^2=\sum_k\left(\sum_{i,j,\alpha}h_{ij}^\alpha\cdot h_{ijk}^\alpha\right)^2\leq\sum_{i,j,\alpha}(h_{ij}^\alpha)^2\cdot\sum_{i,j,k,\alpha}(h_{ijk}^\alpha)^2,$$

that is

$$n^4H^2\|\nabla H\|^2\leq\|h\|^2\cdot\|\nabla h\|^2. \tag{48}$$

From (31) and (48), we have

$$0\geq\int_{M^n}\left\{\|h\|^{-2}n^3(n-1)(c-R)\|\nabla H\|^2+\frac{1}{2}\sum_{i,j,\alpha}R_{ijij}(\lambda_i^\alpha-\lambda_j^\alpha)^2\right\}. \tag{49}$$

Thus, by hypothesis, $\|\nabla H\|^2 = 0$, so H is constant on M^n. Therefore, Theorem 3.3 follows from a result of Aiyama [1] and this completes the proof of Theorem 3.3. □

Acknowledgements

This paper was written during the author's stay at the Max-Planck-Institut für Mathematik in Bonn. The author would like to express his thanks to the institute for its hospitality and financial support. This work is supported in part by the National Natural Science Foundation of China.

References

[1] R. Aiyama, Compact space-like submanifolds in a pseudo-Riemannian sphere $S_p^{m+p}(c)$, Tokyo J. Math. 18 (1995) 81–90.
[2] K. Akutagawa, On space-like hypersurfaces with constant mean curvature in the de Sitter space, Math. Z. 196 (1987) 13–19.
[3] H. Alencar, M.P. do Carmo, Hypersurfaces with constant mean curvature in spheres, Proc. Am. Math. Soc. 120 (1994) 1223–1229.
[4] B.Y. Chen, Geometry of Submanifolds, Marcel Dekker, New York, 1973.
[5] Q.M. Cheng, Complete space-like submanifolds in a de Sitter space with parallel mean curvature vector, Math. Z. 206 (1991) 333–339.
[6] Q.M. Cheng, S. Ishikawa, Space-like hypersurfaces with constant scalar curvature, Manuscripta Math. 95 (1998) 499–505.
[7] S.Y. Cheng, S.T. Yau, Maximal space-like hypersurfaces in the Lorentz–Minkowski space, Ann. Math. 104 (1976) 407–419.
[8] S.Y. Cheng, S.T. Yau, Hypersurfaces with constant scalar curvature, Math. Ann. 225 (1977) 195–204.
[9] A.J. Goddard, Some remarks on the existence of space-like hypersurfaces of constant mean curvature, Math. Cambridge Philos. Soc. 82 (1977) 489–495.
[10] H. Li, Global rigidity theorems of hypersurfaces, Ark. Mat. 35 (1997) 327–351.
[11] L. Ximin, Space-like hypersurfaces of constant scalar curvature in the de Sitter space, Atti Sem. Mat. Fis. Univ. Modela 48 (2000) 99–106.
[12] S. Montiel, An integral inequality for compact space-like hypersurfaces in de Siter space and applications to the case of constant mean curvature, Indiana Univ. Math. J. 37 (1988) 909–917.
[13] B. O'Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic Press, New York, 1983.
[14] M. Okumuru, Hypersurfaces and a pinching problem on the second fundamental tensor, Am. J. Math. 96 (1974) 207–213.
[15] J. Ramanathan, Complete space-like hypersurfaces of constant mean curvature in the de Sitter space, Indiana Univ. Math. J. 36 (1987) 349–359.
[16] Y. Zheng, Space-like hypersurfaces with constant scalar curvature in the de Sitter spaces, Differ. Goem. Appl. 6 (1996) 51–54.
[17] Y. Zheng, On space-like hypersurfaces in the de Sitter spaces, Ann. Glob. Anal. Goem. 13 (1995) 317–321.